Contribution ID: **281** Type: **Presentazione orale**

Rate-Dependent Spherical and Conical Indentation of soft materials with application to biomechanics

Thursday 3 September 2026 15:30 (15 minutes)

Indentation testing is widely used for characterizing the mechanical behavior of soft solids, including elastomers, polymers, hydrogels, and biological tissues. In these materials, the response is strongly time-dependent, with the measured force depending on both the imposed displacement and the complete deformation history. Constitutive interpretation of indentation data becomes particularly challenging when relaxation processes span many decades of time. In such cases, conventional rheological models with only a limited number of discrete relaxation times, such as SLS (Standard linear solid), often fail to capture the broad relaxation spectrum adequately.
The present work adopts a Modified Power-Law (MPL) relaxation kernel, which offers a compact yet highly flexible description of broad-spectrum viscoelasticity while remaining analytically tractable in hereditary integral formulations. This constitutive model supports a unified viscoelastic contact-mechanics framework for rigid spherical and conical indentation of a linear viscoelastic half-space.
For monotonic ramp loading, both spherical and conical problems are solved analytically via the Lee–Radok viscoelastic correspondence principle, employing the classical Hertz and Sneddon elastic solutions as reference kernels. The resulting hereditary integrals provide closed-form force–displacement–time expressions involving Gauss hypergeometric functions for both geometries. A normalized apparent indentation modulus is defined by inverting the elastic contact relations, serving as a compact descriptor of the evolving viscoelastic modulus. The governing equations are recast in dimensionless form, highlighting the essential competition between imposed loading rate and intrinsic relaxation dynamics. A dimensionless loading-rate parameter controls the balance between instantaneous elastic response and viscoelastic dissipation, while the MPL decay exponent governs the breadth and persistence of the relaxation spectrum. Smaller exponent values slow kernel decay, sustaining higher forces and apparent moduli over longer loading durations.
Analytical predictions are validated against axisymmetric finite-element simulations using PLANE183 elements and surface-to-surface contact pairs, reproducing the expected physical trends for both spherical and conical indentation under ramp loading. Higher loading rates shift the response toward the glassy limit with elevated forces and moduli, while slower rates allow greater relaxation toward the rubbery limit. The material exponent modulates relaxation behavior, with lower values preserving elevated response across a wider range of rates.
The analysis is further extended to triangular loading–unloading cycles with identical loading and unloading rates for both indenter geometries. In both cases, the loading stage follows directly from the Lee–Radok formulation, whereas the unloading stage is computed through an approximate Lee–Radok-type split-history treatment of the viscoelastic relaxation integral after the instant of maximum indentation depth. The resulting formulation captures hysteresis, unloading-force relaxation, and detachment trends for both spherical and conical indentation within a common hereditary framework.
Hence, this work address the most common loading conditions in indentation tests of soft materials, commonly adopted for mechanical characterization of biological tissues. It provides a unified viscoelastic contact-mechanics framework that brings together spherical and conical indentation, ramp and triangular histories, closed-form analytical solutions, and finite-element verification under one constitutive description based on the modified power-law model, which has been proved to be an excellent approximation of the material response spectrum over wide range of excitation frequencies. The framework captures the coupled influence of loading rate, unloading rate, and relaxation spectrum on the force response, apparent indentation modulus, and hysteresis. This MPL-based approach provides a robust platform for the mechanical characterization of soft materials and biological tissues, enabling reliable inverse identification of constitutive parameters from rate-dependent indentation experiments.

Primary authors: Mr SHIFERAW, Aleligne Yohannes (Politecnico di Bari); TRICARICO, Michele (Politecnico di Bari); AFFERRANTE, Luciano (Politecnico di Bari); PAPANGELO, Antonio (Politecnico di Bari)

Presenter: PAPANGELO, Antonio (Politecnico di Bari)

Session Classification: Modellazione

Track Classification: Modellazione