

# Experimental Validation of a Numerical Analysis for Transient Resonance Response of an Integral Bladed Disk †

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## Abstract

The design process of modern aeronautical engines requires a deep understanding of the dynamic behaviour of each component during steady and unsteady operating conditions. In particular, the design of rotating components is subjected to a dedicated validation for the dynamic characterization and stress certification by means of specific tests where the vibrations of bladed disks of the compressor and turbine modules are measured in rotating conditions. During spinning test rig tests, predefined acceleration ramps simulate operating conditions. According to the level of validation, the certification of a single bladed disk can be performed using an excitation system that replicates the loads originally due to fluid-structure interaction in the working turbo engine. Experimental Campbell diagrams may show unconventional dynamic responses, referred to as Non-Synchronous Vibrations, not predicted by standard industrial dynamic analyses based on Synchronous vibrations caused by the unsteady flow which excites the single blade with a frequency that is a multiple integer of the rotation speed. In literature, a plausible cause is identified in transient effects introduced by excessive ramps; this work aims to verify if the FE analysis of the transient dynamic response of a perfectly tuned disk is able to effectively predict Non-Synchronous Vibrations measured through a test campaign conducted on a dummy integral bladed disk.

**Keywords:** transient resonance response; Non-Synchronous Vibration; sweep rate; bladed disk; finite element analysis; experimental validation;

## 1. Introduction

The aviation industry is currently facing increasingly complex challenges in designing more efficient, safe, and environmentally friendly aero engines which, not only during service, but also in certification tests, are subjected to acceleration ramps with various sweep rates.

At the same time, the operational flexibility required by power plants is increasing due to a rise in the diffusion of renewable energy worldwide, thus steam and gas turbines need to modulate their output more frequently and rapidly according to the variability of the primary source of energy or users' needs.

Part of the design process of these machines thus involves the execution of a series of dynamic characterization and stress certification tests where, in a spinning test-rig, a predefined acceleration ramp is recreated simulating different operating conditions. Some of these tests are related to the validation of that part of the design that addresses the possible damage caused by Low Cycle Fatigue and creep. In this case the load cycles produced by the centrifugal effect during rotation are taken into account in a series of ramp-up and ramp-down simulating the sequence of more missions of the turbo-engine. The operating loads due to fluid-structure interaction on the bladed disks are more of interest for the present study, since they are the main cause of vibrations and, therefore, High Cycle Fatigue phenomena. In this case, unsteady loads normally generated by the flow passing through the rotating modules can be replicated by a purposely designed excitation system that can work also in vacuum and for a single rotor. It is not rare that, during these tests, unconventional

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dynamic responses (Non-Synchronous Vibrations) are observed, which are not predicted by standard dynamic analysis like flutter analysis.

In literature [2], an explanation has been found in transient effects introduced by frequent and quick resonance passages that can be modelled with a time-varying excitation frequency. This phenomenon can lead to a series of effects that influence not only bladed disks dynamics, but also the overall operational safety (Fig. 1):

- Presence of a beating effect after the primary peak of the response;
- Usually, a decrease in the amplitude of the main peak;
- The persistence of the resonance phenomenon for a wider interval of the rotational speed during sweep;
- Shifting of the location of the transient response in the direction of the sweep.

Observing the transient response in the time domain, it becomes clear that, if these effects are ignored, the HCF life estimation, based on the number of cycles that exceed a certain mean stress threshold, becomes inadequate due to the prolonged presence of beats in the oscillation, affecting the correct design process. Moreover, the identification of modal parameters, like the Half Power Band Method (HPB) for the determination of the modal damping ratio or the natural frequency, has poor accuracy leading to a large systematic error associated with the determination of their values, as will be shown, due to the listed problems.

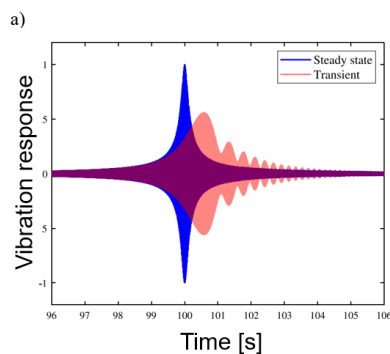


Figure 1: Time history of a vibration response in stationary and transient condition [3].

The work, developed in collaboration with GE Avio, aims to understand whether commonly adopted commercial Finite Element analysis tools can be efficiently used to accurately predict this behavior in the design phase.

Traditionally, complex dynamic analyses on real systems are conducted exploiting the cyclic symmetry property, which bladed disks possess due to their geometry; a fundamental (or basic) sector, can be in fact identified that, if patterned circularly about an axis of symmetry, represents the full component. This standard approach inherently assumes a perfectly "tuned" system, in which every sector is structurally and dynamically identical. However, actual bladed disks inevitably exhibit small, random variations in their geometric and material properties due to manufacturing tolerances, material flaws, and assembly procedures.

This unavoidable phenomenon, widely known as *mistuning*, breaks the ideal cyclic symmetry of the rotor, and influences the dynamic behavior of bladed disks through:

- Mode shapes distortion;
- Response localization;
- Complex modes' decoupling.

The objective is thus to quantify the deviations introduced by these inherent structural asymmetries and determine whether the tuned-system approximation remains a sensible and safe approach during the design phase.

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## 2. Theoretical background and methodology

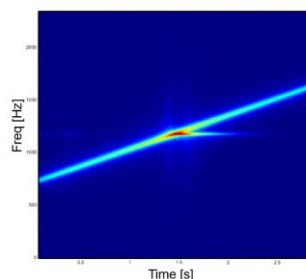
An integrated theoretical, experimental, and numerical methodology is adopted to evaluate the predictive capability of finite element (FE) transient analyses for bladed disks. The approach follows four sequential phases: establishing the theoretical framework of transient resonance crossings and Non-Synchronous Vibrations; executing a rotational test campaign on a dummy blisk across various sweep rates; developing and calibrating a tuned FE model using non-rotating modal data; and quantitatively validating the transient numerical simulations against experimental strain-gauge measurements.

### 2.1 Transient resonance response and Non-Synchronous Vibrations

For a specific mode of vibration, the homogeneous response represents a transitory element that decays in accordance with the exponential decay function, dependent on the modal damping ratio. For a steady-state or quasi steady-state operation, the sweep velocity is slow enough that this component is dissipated and does not interact with the forced response and as such no transient effects are activated. Above a critical value of acceleration rate, on the contrary, the excitation frequency change is comparable to the decay time of the natural frequency response, thus a superposition of two signals with close frequencies occurs, generating the beating phenomenon, and the existence of a second trace, that of the free-decaying vibration.

By examining the frequency content of a transient resonance response, like the one displayed in Fig. 2, it is possible to notice the coexistence of two components [2]. The first frequency content is associated to the synchronous vibration whose value is a multiple integer time of the rotation speed. This response is typical of the Engine Order (EO) response that is also found in a quasi-static crossing (low acceleration or, in other words, slow ramp) of a resonance; however, what distinguishes the transient response from a quasi-static crossing is the presence of a second vibratory component, a visible homogeneous response is in fact triggered along the natural frequency of the system; this is visible in a Campbell's diagram as a horizontal trace that diverges from the EO response (Fig.2). This vibratory component of the response corresponds to the transient resonance crossing producing a Non-Synchronous Vibration, i.e., not strictly correlated to the rotation speed of the engine. The transient response decays in accordance with the exponential decay function, mainly depending on the modal damping ratio.

If the sweep rate is large enough, i.e., a fast ramp is imposed, this harmonic component interacts with the engine order response; since the two vibrations are characterized by similar frequency content, beating phenomena arises.



**Figure 2:** Campbell's diagram of a Single Degree of Freedom system's transient resonance crossing.

The other effects can be explained considering that, due to a high sweep velocity the time available to store energy into the system near the resonance is reduced, lowering the maximum amplitude of the response in a non-stationary condition, while the frequency shift is caused by a delay with respect to the resonance condition. According to [2], the intensity of these effects, in absence of nonlinearities, and for a specific mode of vibration of a complex system, depends on the value of a single dimensionless value called Reduced Sweep Velocity:

$$K = \frac{\dot{f}}{\zeta^2 f_n^2} \quad (1)$$

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Se vuoi mantenere la sezione 2 come metodologia, inizia con un breve cappello dove spieghi cosa intendi per metodologia (prima teoria poi pratica poi simulazione numerica) con un filo logico

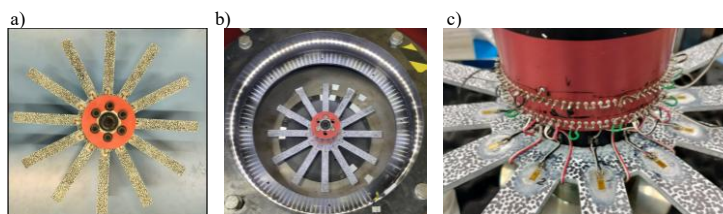
This includes, as expected, the frequency rate of change of the forcing function (which is linked to the sweep rate of the disk velocity) and the damping ratio and natural frequency of the mode considered. Beating phenomenon is negligible for values of  $K \lesssim 3.6$ .

Brinkmann, Hoffmann, Panning-Von Scheidt and Stür [5] conducted a similar test campaign to the one object of this study and, using the same definition of Reduced Sweep Velocity, they found observed beating in the measured linear responses for values of  $K$  higher than the cited limit, as expected.

## 2.2. Test campaign setup

In order to verify the capability of commercial FEA software to accurately predict the transient resonance response of a rotating disk, a test campaign has been conducted to produce an experimental database to compare with simulations.

The test article is a dummy integral bladed disk, realized in an aluminum alloy, characterized by 12 beams representing the blades. At the tip of each blade a cylindrical neodymium (NdFeB) magnet is inserted to allow for a localized non-contact excitation during the rotation. The magnetic axis of the magnets is parallel to the rotation axis (Fig.3).



**Figure 3:** (a) Test Article. (b) Spinning test rig. (c) Strain gauges placement and connection to the telemetry

The disk is mounted on a vertical rotating shaft of an electrical motor mounted in a spinning test rig housed in a vacuum chamber to remove possible aerodynamic excitations with turbulent air surrounding the disk. In order to produce the EO excitation (Synchronous vibrations), permanent magnets are equally spaced along the circumference of the static flange support of the rig and are mounted with the magnetic axis also parallel to the rotation axis. The fixed magnets interact with the magnets on the tip of the dummy blades according to an axial repulsive force. The number of the fixed magnets determines what EO value that are generated on the disk. For these tests, to excite all the EOs inside the speed range, only one fixed magnet has been used as the force applied on the disk hitting all the blades in a localized circumferential position can be approximated to an impulsive load whose harmonic content is wide and covers all the EOs.

To measure the dynamic response, strain gauges have been placed in a half-bridge configuration on each blade's root, where strain values are the highest for all the *Nodal Diameters*' (NDs) mode shapes of the first modal family (1<sup>st</sup> Flexural Out of Plane – OOP – mode). Given a mode shape of a mechanical system characterized by cyclic symmetry property (tuned system), the nodal diameter is a nodal line passing through the center where the modal displacement is zero, and across which adjacent sectors of the symmetric structure respond out of phase [1]. A telemetry system by *Manner* has been used to transmit strain gauges analog data to the acquisition system, a *National Instrument* A/D converter. The telemetry has a voltage measurement range of  $- / +10V$ ; gain and offset on each bridge have been set to fall into this range for a maximum tip displacement of  $\sim 1mm$  through a static calibration of the strain gauges. A known force has been applied on the tip of each blade, precisely on the permanent magnets that receive the repulsive force during the spin test, via a steel block of the weight of 0.99 kg positioned using a permanent magnet of 0.01 kg. The voltage given is then adjusted to have a maximum reading for the desired tip displacement.

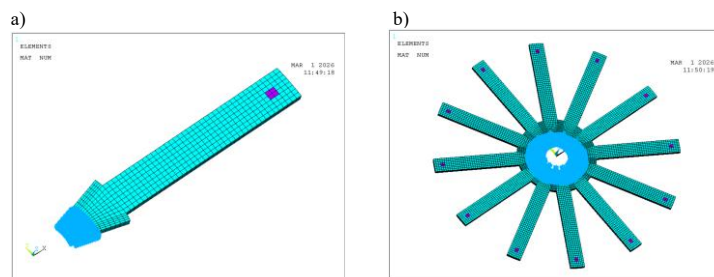
## 2.3. Finite Element model of the bladed disk

A single sector representative of the full structure can be identified thanks to the geometry of the blisk and the constraints applied to it; this basic sector (Fig. 4(a)) is composed of a single blade and a portion of the disk, corresponding to an angle of 30 degrees of the full model (Fig. 4(b)).

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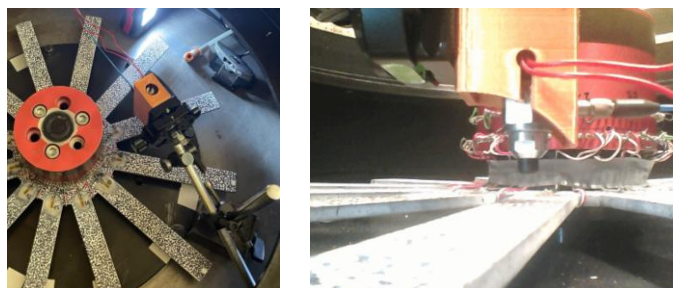
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The domain has been discretized using 3D 20-nodes solid element SOLID 95 using Ansys Mechanical APDL 2022 R2 with two different material models for the aluminum and the permanent magnet; to replicate the constraints generated by the bolting connections at the disk hub that include the telemetry antenna, all translations and rotations on selected nodes were set to zero. The complete disk is obtained by duplicating the base sector.



**Figure 4:** (a) Basic sector. (b) Full model.

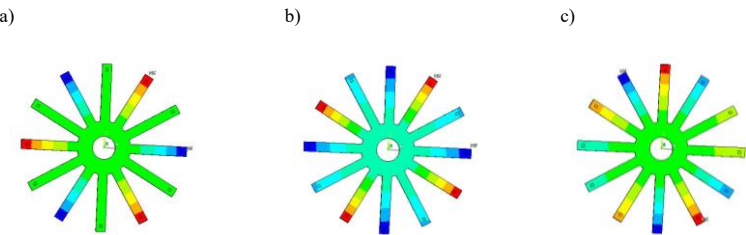
The model has been tuned with respect to the first modal family of the blisk (1<sup>st</sup> Flexural OOP mode) obtained from a non-rotating hammer test in vacuum condition. For this reason, an automatic hammer has been developed at AerMec Laboratory at Politecnico di Torino (Firrone, C. M.; Battiatto, G.; Usmanov, U. (2025). *Sistema autonomo per Hammer Test nell'analisi modale sperimentale*. Knowledge Share No. 102025000001362) and positioned inside the vacuum spinning rig (Fig. 5). A remote control based on Arduino platform produced the optimal voltage and time of activation of the linear actuator of the automatic hammer device in order to execute the hammer test. The automatic hammer is equipped with a load cell in order to obtain the Frequency Response Function (FRF) of the system. The response was measured simultaneously with the strain gauges already applied on the disk and one accelerometer placed along one of the blade length that will be removed during the spinning test. The two measurement systems were adopted for a more accurate update of the FE model since two different response locations have been used. Furthermore, separate measurements were also performed to verify that the accelerometer does not add a mass which is relevant to perturb the disk response.



**Figure 5:** Automatic hammer (orange box) positioned over one dummy blade: top view (left) and side view (right).

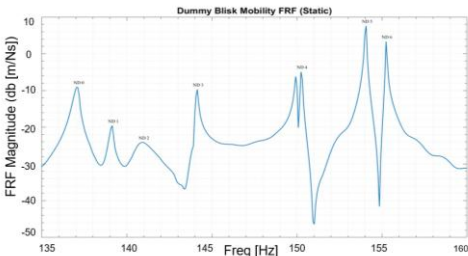
The FRF response is shown in Fig. 7. The resonance frequency and the damping of the first modes belonging to the first modal family (OOP motion of the blades) have been identified. The FE model has been updated correspondingly by refining the Young's modulus of the disk material and the constraint at the hub. For the comparison between numerical simulations and experimental measurements the last modes of Fig.7 have been considered, namely those characterized with the Nodal Diameters (NDs) 3, 4 and 5. The minimum number of nodal diameters is 0 while the maximum value  $ND_{max}$  is equal to the number of blades  $Z$  divided by 2 (if  $Z$  is even). If a mode shape has a number of nodal diameters greater than 1, they are equally spaced around

the circumferential direction as shown in Fig. 6 where the FE modal analysis result is shown for the mode shapes of interest with their typical wavy deflection shape around the circumferential direction.



**Figure 6:** FE mode shapes characterized by ND=3 (a), ND=4 (b), ND=5 (c), modal axial displacement is shown.

In particular, the mode shapes characterized by a value of nodal diameter comprised between 0 and  $ND_{max}$  are repeated, i.e., for the same natural frequency two identical mode shapes exist in quadrature (where one of the modes has a maximum or minimum modal displacement, the second shows a nodal diameter with null modal displacement). If the system is not perfectly cyclically symmetric, the repeated modes are characterized by a slightly different natural frequency. This is what happens in reality because real bladed disks are always characterized by slightly different sectors due to machining tolerances, assembly uncertainties, and random differences caused by wear in service. This effect, which determines a mistuned bladed disk, is also clearly visible in Fig.7 for the repeated mode shapes characterized by ND=4 where the frequency response function of the test article shows clear evidence of mistuning. When mistuning is not clearly visible in the FRF, the modal damping of the two repeated mode shapes is large enough to hide the presence of the close resonances pair. Conversely, because the present Finite Element model was constructed by duplicating a single sector, it remains tuned, and thus, without introducing intentional asymmetries into the model, it cannot exactly replicate the real bladed disk's response, thus limiting the accuracy of the numerical analyses.



**Figure 7:** Experimental blisk Mobility FRF

**Table 1.** Resonance frequencies comparison between hammer test and FEM for the modes used in the validation.

| Nodal Diameter | Frequency – Exp. [Hz] | Frequency - FEM [Hz] | Delta [%] |
|----------------|-----------------------|----------------------|-----------|
| 3              | 144.14                | 144.57               | -0.298    |
| 4              | 150.23                | 149.28               | 0.630     |
| 5              | 154.06                | 152.58               | 0.963     |

At the end of the tuning procedure, the difference between the values of the FE natural frequencies and those of the experimental test is below 1% for the modes of interest, thus showing a good correlation for the purpose of this work.

#### 2.4. Transient dynamic analysis in Ansys Mechanical APDL

In Ansys Mechanical APDL two methods are available to perform a transient dynamic analysis: full and mode-superposition. The full method uses the full system mass, stiffness and damping matrices to calculate the transient response; it is the more general method and enables all types of nonlinearities to be included. Its main disadvantage is that it is more computationally intensive than the mode-superposition analysis. The mode-superposition method, instead, uses the natural frequencies and mode shapes of a linear structure, extracted through a previous modal analysis, to predict the response to transient loads. At this stage of the work, typical nonlinearities encountered in bladed disk dynamics, such as contact and friction damping, have been excluded, and consequently, the analyses belong to the class of linear problems, and the mode-superposition method can be adopted. The damping matrix is generated using the modal damping ratios that are identified from the hammer test.

To recreate a travelling-wave type of excitation, in a cylindrical reference frame fixed on the disk, a series of sinusoidal functions were applied on each blade tip. The phase angle between adjacent blades depends on the EO of the excitation; the linear sweep is instead simulated by introducing a constant acceleration. For the  $i$ -th blade the harmonic loading, along the z-axial direction, is:

$$F_{zi} = \widehat{F}_{zi} \cdot \sin \left( 2\pi \cdot \left( \frac{1}{2} \dot{f} t^2 + f_0 t \right) + \psi_i \right) \quad (2)$$

Where:

- $\widehat{F}_{zi}$  is the amplitude of the forcing function in N;
- $\dot{f}$  is the sweep rate in Hz/s;
- $f_0$  is the starting frequency in Hz;
- $\psi_i$  is the phase angle of the  $i$ -th blade.

The phase angle  $\psi_i$  is linked to the EO by the relationship:

$$\psi_i = EO \frac{2\pi}{N_{tot}} (i - 1) \quad (3)$$

With  $N_{tot}$  the total number of blades.

These time-dependent loadings are applied as nodal forces through tabular input by defining a one-dimensional table where the rows are identified by time and the single column represents the force value in each time step. The nodes chosen for the loading are those at the center of the permanent magnets at the tip of each dummy blade.

#### 2.5. Test campaign in rotating condition

A total of six tests were performed with different sweep rates, strain-gauge sampling frequencies and directions of sweep (acceleration – sweep up, and deceleration – sweep down). The campaign was organized in order to observe the influence of the sweep velocity on the measured response and to select a test that is suitable for the validation of the numerical model.

Table 2. December 2025 test campaign.

| Test name | Sweep rate [RPM/s] | SG sampling frequency [Hz] | Direction |
|-----------|--------------------|----------------------------|-----------|
| 1         | 1                  | 1000                       | Up        |
| 2         | 16                 | 1000                       | Up        |
| 3         | 16                 | 14000                      | Up        |
| 4         | 16                 | 14000                      | Down      |
| 5         | 64                 | 14000                      | Up        |
| 6         | 64                 | 14000                      | Down      |

For the validation presented in this paper, Test 3 (Fig. 8) was chosen because it provides significant transient effects, while still maintaining sufficient separation between adjacent EOs responses, unlike tests with higher acceleration where overlapping of the responses at different EO occurs.

3. Experimental validation of FEM transient analyses

To verify the capability of the FE model to accurately predict the transient resonance response of the dummy blisk, the third test from the test campaign was chosen. During Test 3, the shaft speed was ramped up from circa 400 RPM to 1600 RPM with a sweep rate of 16 RPM/s; in this speed range, because of the presence of a single magnet mounted on the static ring, all the nodal diameters of the disk were excited.

From the Campbell diagram of Test 3, EOs 7, 8 and 9 were selected since they show a clear non-synchronous vibration response, indicating a strong non-stationary response.

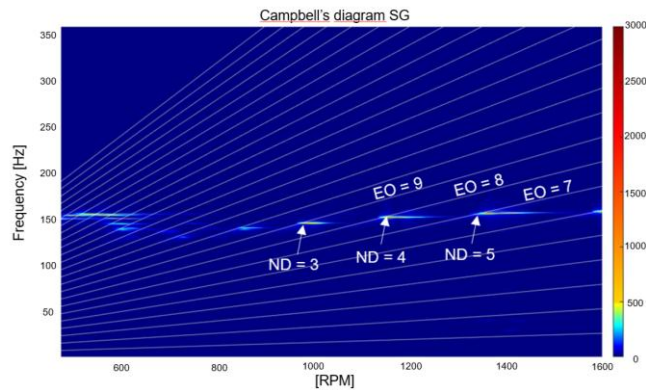


Figure 8: Campbell diagram from Test 3

Resonances occur at the intersections with three distinct modes, each characterized by a specific nodal diameter, respectively: ND = 5, 4 and 3.

Since these crossings take place at different rotational speeds, the magnitude of the centrifugal stiffening effect varies for each excited mode. This leads to a mode-specific shift in the resonance frequencies that has to be taken into account in the simulation via a pre-stressed modal analysis, conducted specifically for each ND, as the starting point for the following transient dynamic analysis. The resulting natural frequencies are once again compared in Table 3 to the experimental one to confirm the accurate tuning of the FE model even after the introduction of the centrifugal stiffening effect.

**Table 3.** Pre-stressed modal analysis results.

|                       | ND 3     | ND 4     | ND 5     |
|-----------------------|----------|----------|----------|
| Campbell [Hz]         | 146.5    | 153.0    | 157.5    |
| Ansys Mechanical [Hz] | 146.1    | 151.4    | 155.4    |
| Damping ratio         | 5.32E-04 | 5.03E-04 | 1.75E-04 |
| Difference [%]        | 0.27     | 1.05     | 1.34     |

The difference between the values of the two sets of natural frequencies increased in presence of inertia loads, while still remaining under 2%.

However, it must be noted that the increased difference can be explained for the frequency shift of the experimental transient response (as shown in the example of Fig.1) that cannot be included in the pre-stressed modal analysis in the FE modal analysis. As a consequence, the values of the Campbell resonance frequency can be considered as an approximation of the actual natural frequencies. In particular, it can be calculated that the Reduced Sweep Velocity  $K$  for the studied cases is greater than 3.6, even for the minimum sweep rate of 1 RPM/s, coherently with the transient response evidence.

### 3.1. Validation results

The results of the transient mode-superposition simulations for each of the three studied modes were compared with the data recorded by the strain gauges. The experimental and numerical time histories are overlaid on a global x-axis that represents the time from the start of the test. The objective is to evaluate the effectiveness of the FEM to predict the magnitude of transient effects in the response, in particular the time at which the resonance is crossed, the magnitude of the primary peak, the ratio between primary and secondary peaks and the duration of the beating effect, when present. To ensure consistency with the FEM displacement, the voltages given by the strain gauges were converted into strain, and then millimeters, using the FE model. Therefore 12 calibration factors [m/V] were obtained for each strain gauges half bridge assembly.

It is important to highlight that, while during the experimental sweep all the EO excitations are present and all the modes are excited, in the simulation only one travelling-wave response per time can be obtained since the travelling force of Eq.2 is specific for one EO value only.

The time history extracted from SGs applied to the first blade will be shown as a representative comparison for all the blades. Fig. 9-11 show the numerical/experimental comparison in the range of interest for the selected modes. The analyses show that transient effects are clearly present, as expected for the chosen sweep rate. The most prominent aspect emerging from these plots is a mismatch in the time at which the resonance occurs and therefore in the position of the main peak.

This mismatch can be explained by the difference in the natural frequency between the real disk and the FE counterpart: the pre-stressed modal analysis testifies a difference of circa 0.3% to 1.5% (Table 3) with respect to the measured values from the experimental Campbell diagram. This difference expressed in terms of Hertz corresponds to a difference in terms of seconds at which critical crossings occur.

However, the error associated to the time at which resonance peak occurs is always below 3%. The second step of the validation consists in the comparison of the beating response between the two datasets for each nodal diameter, without considering the time shift.

The response at ND = 5 (Fig. 9) shows a very good fitting with the strain-gauge signal and the ability to correctly predict the amplitude of the series of peaks, with an error lower than 1% for the primary and secondary peaks.

The fitting is less accurate for ND 4 (Fig. 10), even less for ND 3 (Fig. 11). These modes show a faster decay in the amplitude of the response compared to the FEM results, at least near the main peak, followed by a sequence of beats with an almost constant magnitude. The experimental response observed in SG 1 for ND = 4 and 3 is mainly caused by the mistuning associated to these modes.

In mistuned systems, the splitting of eigenfrequencies creates adjacent modes with closely spaced frequencies, causing two distinct beating phenomena to overlap and introduce an additional modulation to the dynamic response [10]. This superposition of split frequencies is also the primary mechanism driving the transient amplitude magnification of mistuned systems (TAMS effect) during resonance crossings. Under certain conditions, this interaction amplifies peak transient responses beyond steady-state levels, with the severity of the amplification directly governed by the degree of mistuning [5]. The tuned FE model is unable to replicate this effect.

The summary of the comparisons is reported in Table 4 and 5.

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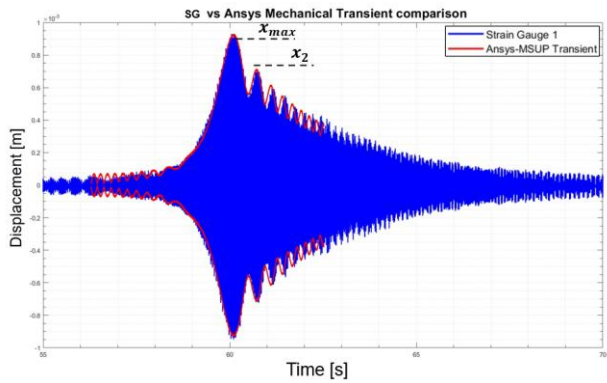


Figure 9: Time-history comparison between FEM and strain-gauge signal for EO 7 and ND 5

Table 4. Comparison results for EO 7 and ND 5.

| Quantity                     | EO 7 - ND 5 SG1 | EO 7 - ND 5 numerical | Error [%] |
|------------------------------|-----------------|-----------------------|-----------|
| Load scale                   | -               | 22.5                  | -         |
| Calibration factor [m/V]     | 0.0003184       | -                     | -         |
| Primary peak position [s]    | 60.1            | 58.4                  | 2.80      |
| Secondary peak position [s]  | 60.8            | 59.0                  | 2.96      |
| Primary peak amplitude [m]   | 9.28E-04        | 9.30E-04              | 0.21      |
| Secondary peak amplitude [m] | 7.10E-04        | 7.14E-04              | 0.56      |
| Secondary/primary ratio      | 0.765           | 0.768                 | 0.35      |

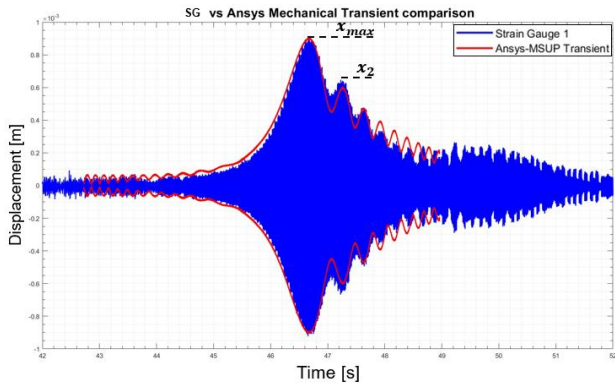


Figure 10: Time-history comparison between FEM and strain-gauge signal for EO 8 and ND 4.

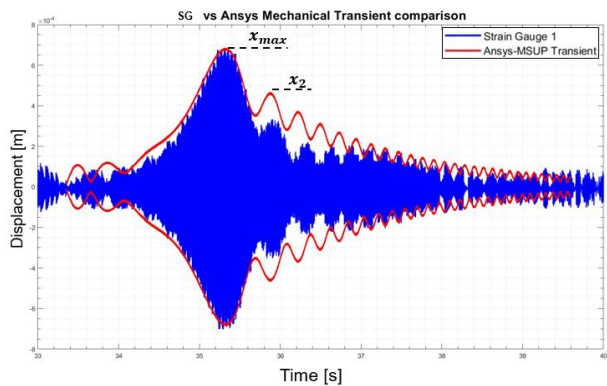


Figure 11: Time-history comparison between FEM and strain-gauge signal for EO 9 and ND 3.

Table 5. Comparison results for EO 8 - ND 4 and EO 9 - ND 3.

| Quantity                     | EO 8 - ND 4 SG1 | EO 8 - ND 4 numerical | Error [%] | EO 9 - ND 3 SG1 | EO 9 - ND 3 numerical | Error [%] |
|------------------------------|-----------------|-----------------------|-----------|-----------------|-----------------------|-----------|
| Load scale                   | -               | 19.8                  | -         | -               | 25.5                  | -         |
| Calibration factor [m/V]     | 0.0003309       | -                     | -         | 0.0003578       | -                     | -         |
| Primary peak position [s]    | 46.7            | 46.0                  | 1.50      | 35.3            | 35.9                  | 1.69      |
| Secondary peak position [s]  | 47.3            | 46.6                  | 1.48      | 35.9            | 36.4                  | 1.39      |
| Primary peak amplitude [m]   | 8.97E-04        | 9.05E-04              | 0.89      | 6.85E-04        | 6.85E-04              | 0.01      |
| Secondary peak amplitude [m] | 6.47E-04        | 6.05E-04              | 6.49      | 3.32E-04        | 4.67E-04              | 40.7      |
| Secondary/primary ratio      | 0.721           | 0.669                 | 7.21      | 0.485           | 0.682                 | 40.6      |

4. Conclusions and future works

The objective of this work is to verify the capability of a commercial finite element software, Ansys Mechanical APDL, to reproduce the transient resonance response observed experimentally on a dummy integral bladed disk subjected to run-up tests. The analysis was based on a workflow that starts from the tuning of the FE model on non-rotating experimental modal analysis with a hammer test, then it proceeds with the modelling of a full 360-degree model for transient dynamic analyses, and finally it concludes with the comparison between numerical and strain-gauge time histories in terms of displacements.

The validation on Test 3 showed a strong correlation between numerical and experimental response for the tuned mode, ND 5 excited by EO 7. In this case the errors on the primary and secondary peak amplitudes were lower than 1%, confirming that the selected modelling strategy is able to capture the magnitude of the transient effects when the real disk behaves sufficiently close to the tuned numerical model.

The comparison decreases in presence of mistuning, as observed for ND 4 and especially ND 3. The tuned FE model cannot reproduce the doublet splitting of repeated modes and the additional transient effects that appear in the real test article, including phenomena such as TAMS. The differences between predicted and measured results are therefore mainly attributable to the difference in the natural frequencies of the selected modes between the real disk and the finite element model, to the approximated damping used in the simulations, and to the actual mistuned geometry of the disk.

In conclusion, for the test cases considered, commercially available FEA software proved to be able to accurately predict the transient response in bladed disks, as well as the related displacement fields for fatigue-damage prediction, provided that the model is correctly tuned and the analyzed mode is not strongly affected by mistuning. Future work should address mistuned numerical models and nonlinear configurations including friction damping to model joints in the assembly.

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